Analysis of Quasi-Periodic Effect in the Design of the Nanorod Metasurfaces

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***Abstract*—Periodic boundary condition is commonly implemented in element simulation for metasurface design. However, the actual boundary condition deviates from this assumption. This results in phase error from the design. This paper offers a theoretical analysis of the quasi-periodic effect of the nanorod metasurface. Efficiency and element phase error of the metasurface are both studied to evaluate the quasi-periodic effect. The result shows that quasi-periodic effect can induce phase error up to 80 degrees, and efficiency of the metasurface decreases up to 40%.**

***Index Terms*—Quasi-Periodic Effect, Metasurface Design, Nanorods Metasurface.**

I. Introduction

Metasurface has recently attracted much attention. The novel feature of complete phase control brings possibilities for some innovative optical elements such as high-efficiency hologram [1] and ultrathin flat lenses [2-4]. The geometric metasurfaces are widely employed due to its superior phase control property [5]. The efficiency of cross circular polarization is studied in [1], which shows that the phase difference between two orthogonal directions is important.

The design of metasurface requires computer-based simulation of the phase-shift property of elements, which is usually proceeded under periodic boundary conditions. However, this assumption could introduce error because the actual elements are only similar but not the same. This is known as the quasi-periodic effect. Reference [6] shows that quasi-periodic effect could induce phase error up to 30 degrees in the design of reflectarray antennas in microwave regime. Metasurfaces in optical regime may have similar effects, but it is not considered in [1].

This study analyzes the element phase error due to quasi-periodic effect in the nanorods metasurface. This paper is structured as follows: Section II outlines the theoretical analysis of the nanorods structure; Section III computes the reflection coefficient for circularly polarized light under periodic boundary; Section IV discusses the element phase-error and efficiency of the metasurface due to quasi-periodic effect.

II. Modelling

The nanorod element studied in [1] is shown in Fig. 1, a 4-layer compound structure is designed to control the phase of reflected light. The phase-shift of reflected circularly polarized light is shown to be linear to the rotating angle of the element.

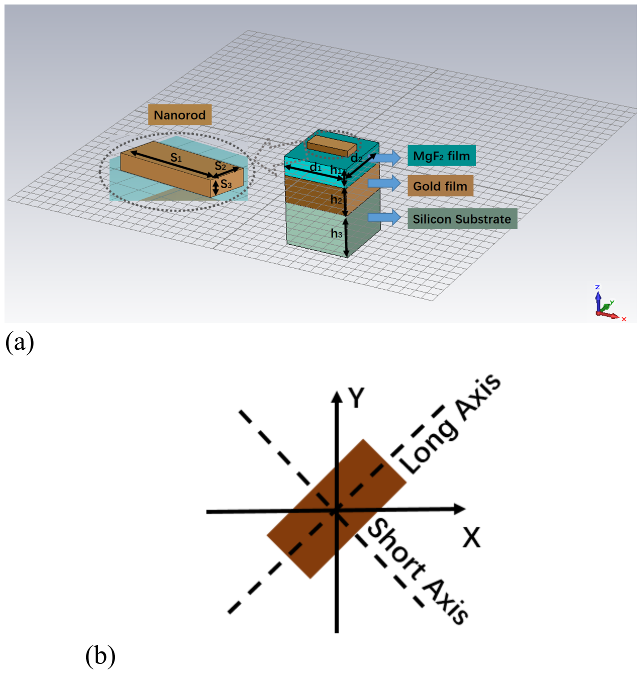


Fig. 1 Element structure of the nanorod metasurface (a) layer structure (b)top view of the nanorod.

A. The Reflection Matrix

The electric field of reflected light follows

$$\begin{bmatrix} E_{r1} \\ E_{r2} \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \begin{bmatrix} E_{i1} \\ E_{i2} \end{bmatrix}, \quad (1)$$

where 1, 2 represents two orthogonal axes, r_{11} and r_{22} are reflection coefficient of co-polarization factor along the two orthogonal axes, and r_{12} and r_{21} are cross-polarization factor; $E_{i1,2}$ represents electric field of the incident light and $E_{r1,2}$ represents the electric field of the reflected light.

The four matrix-element represent the reflection coefficient of co-polarization and cross-polarization of two linearly polarized light with orthogonal polarization along the two axes. For instance, when the polarization of incident light is along Direction 1, the co-polarization reflection coefficient would be with r_{11}, and cross-polarization reflection coefficient would be r_{12}.

B. Reflection of the Nanorod Metasurface Element

As shown in Fig.1, the nanorod element is rotated with an angle θ. Setting the reflection matrix in the nanorod's coordinate (choosing the long axis and the short axis as Direction 1 and 2) as $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, the reflection matrix in the xy −coordinate can be written as

$$\begin{bmatrix} E_{rx} \\ E_{ry} \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} E_{ix} \\ E_{iy} \end{bmatrix} \quad (2)$$

With circularly polarized light

$$\begin{bmatrix} E_{ix} \\ E_{iy} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ \pm j \end{bmatrix} e^{j\omega t}, \quad (3)$$

where ω is the angular frequency, $\pm$ represents right-handed or left-handed circularly-polarized light, (2) becomes

$$\begin{bmatrix} E_{rx} \\ E_{ry} \end{bmatrix} = \frac{1}{2}(A_{11} + A_{22}) \begin{bmatrix} 1 \\ \pm j \end{bmatrix} e^{j\omega t} + \frac{1}{2}[(A_{11} - A_{22}) + (A_{12} + A_{21})j]e^{-j2\theta} \begin{bmatrix} 1 \\ \mp j \end{bmatrix} e^{j\omega t} \quad (4)$$

The reflected field contains both left-handed and right-handed circularly-polarized components. The coefficients of co-polarization and cross-polarization are

$$r_{co} = \frac{1}{2}(A_{11} + A_{22}) \quad (5)$$

$$r_{cross} = \frac{1}{2}[(A_{11} - A_{22}) + (A_{12} + A_{21})j]e^{-j2\theta} \quad (6)$$

It can be observed from (5) that when $A_{11} = -A_{22}$, $r_{co} = 0$, which means the co-polarized field varnishes when the reflection coefficient along the long axis and the short axis have the same amplitude and a phase difference of π.

The phase shift of the cross-polarized reflected field is

$$\varphi = angle\big((A_{11} - A_{22}) + (A_{12} + A_{21})j\big) - 2\theta, \quad (7)$$

which shows that when $A_{11,12,21,22}$ are constant, the phase of r_{cross} is linear to θ.

The cross-polarization efficiency is

$$\mu = r_{cross} r_{cross}^{*}. \quad (8)$$

According to (5) and (6)

$$\mu = \frac{1}{4}|(A_{11} - A_{22}) + (A_{12} + A_{21})j|^2 \quad (9)$$

III. Properties under Periodical Boundary

The model of nanorod element structure shown in Fig. 1(a) is simulated by CST® under periodic boundary condition. The element parameters marked in Fig. 1(a) are listed in Table 1.

TABLE I. PARAMETERS OF NANOROD STRUCTURE

Description	*Parameter*	*Value(nm)*
Side length of the base	d_1, d_2	300
MgF2 film thickness	h1	90
Gold film thickness	h2	130
Silicon substrate thickness	h3	200
Length of the nanorod	s1	200
Width of the nanorod	s2	80
Height of the nanorod	s3	30

A. Calculation of Reflection Coefficient

The reflected electric field right above the element with linearly polarized incident light is shown in Fig.2, where (a) and (b) are co- and cross-polarized reflected field while the incident field is polarized along the long axis; (c) and (d) are co- and cross-polarized reflected field while the incident field is polarized along the short axis.

The reflection coefficient for linearly polarized light is

$$A = \frac{\iint E_r ds}{\iint E_i ds} \quad (11)$$

where E_r and E_i are electric field of reflected and incident light. This formula can be derived based on Floquet theorem [6].

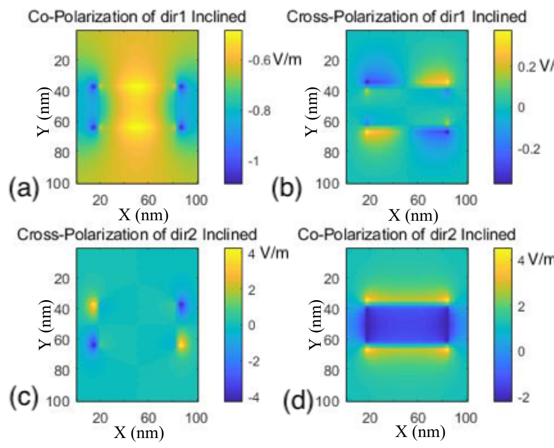
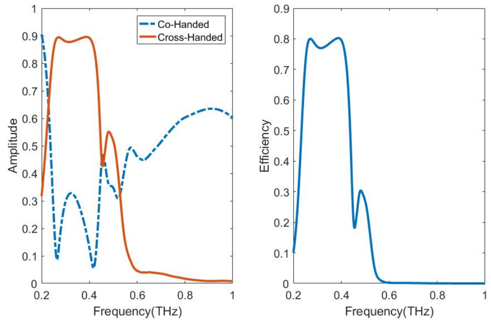


Fig. 2 Plot of the reflected electric field above the element.

B. Computaton of Reflection Coefficient and Efficiency

The reflection coefficient and efficiency can be computed by (7)-(9), as shown in Fig. 3. An efficiency over 70% between 0.249THz and 0.423THz can be achieved. The element simulation is done under periodic boundary condition.



(a) Reflection Coefficient (b) Efficiency

Fig. 3 Element reflection coefficient and efficiency of circular polarization with frequency.

C. *Reflection Phase under Different Rotating Angle*

As shown in (7), the phase-shift of reflected field is linear to the rotating angle. As shown in Fig. 4, high linearity can be acquired at 0.375THz.

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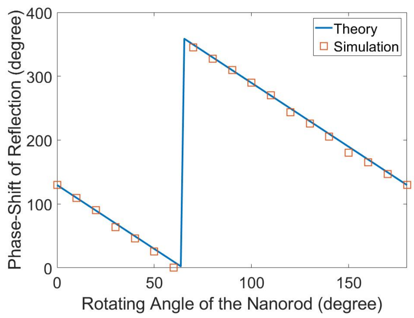

Fig. 4 Phase-shift of circularly cross-polarized reflection under different rotating angle.

IV. Quasi-Periodic Effect

Fig. 3 and Fig. 4 show that we can achieve good phase-control property and over 70% efficiency based on element simulation under periodic boundary condition. In this section, we will study the quasi-periodic effect due to the difference among neighboring elements at 0.375THz. As shown in Fig. 5, the central element is surrounded by 8 elements. These eight elements are same but rotated by an angle α to their long axis.

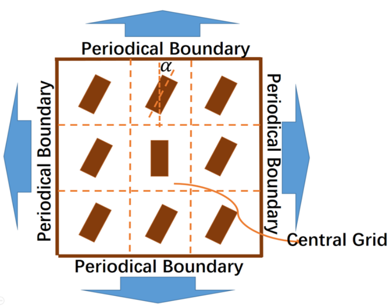

Fig. 5 3x3 quasi-periodical cell array.

The reflected electric field right above the central element when the boundary elements are rotated 90 degrees is shown in Fig. 6. Obviously, the central element is strongly coupled with its neighbors. This coupling affects both reflection phase and efficiency.

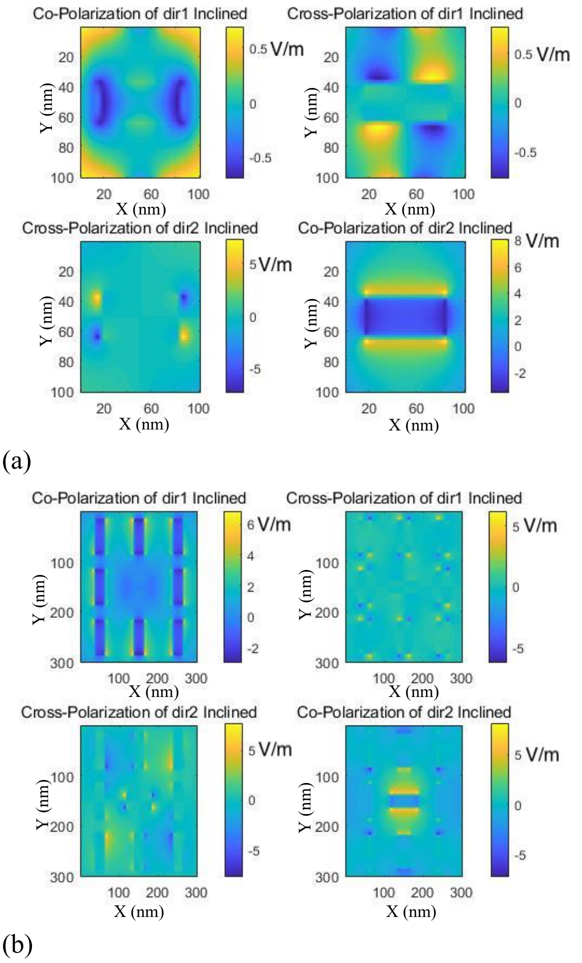

Fig. 6 Plot of electric field above the element (a) central element (b). 3x3 element cell array.

A. *Calculation of Phase Error*

The actual element phase would be no long linear to θ when the periodic boundary condition is broken. We can calculate the phase error

$$\delta = \varphi_{PB} - \varphi_{QB} \tag{10}$$

where φ_{PB} is the element phase shift under periodic boundary computed in Sec. III, and φ_{QB} is the element phase shift computed in Sec. IV where the surrounded elements are different from the central element.

The reflection coefficients of linearly polarized light are shown in Fig. 7. The increase of cross polarization A_{12} and A_{21} would result in phase error for circularly polarized light.

The phase error of circularly polarized light on the central element calculated by (10) is shown in Fig. 8. This error is up to 60 degrees when the angle of boundary elements is close to 90 degrees.

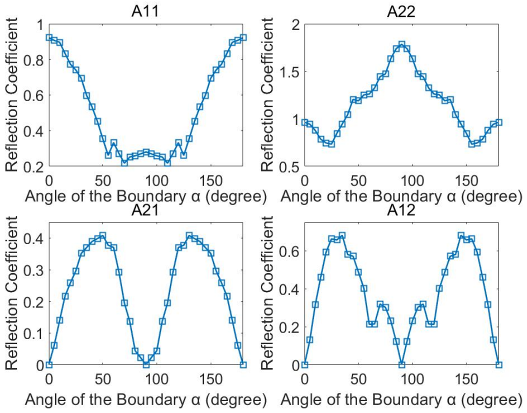

Fig. 7 Reflection coefficient of linear polarization.

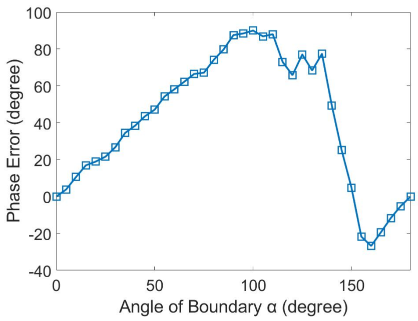

Fig. 8 Phase error of circular polarization induced by quasi-periodical effect.

B. *Influence to Efficiency*

The reflection coefficient for circularly polarized light can be computed by (5) and (6). As shown in Fig. 9, the quasi-periodic effect causes rise in co-polarization and decrease in cross-polarization. This reduces the efficiency. As shown in Fig. 10, the efficiency decreases up to 40% with the angle α close to 90 degrees, which means the inconsistency among the elements could cause a decrease in the cross-polarization.

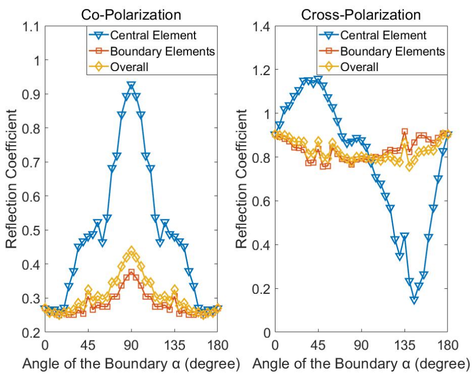

Fig. 9 Reflection coefficient for circular polarization.

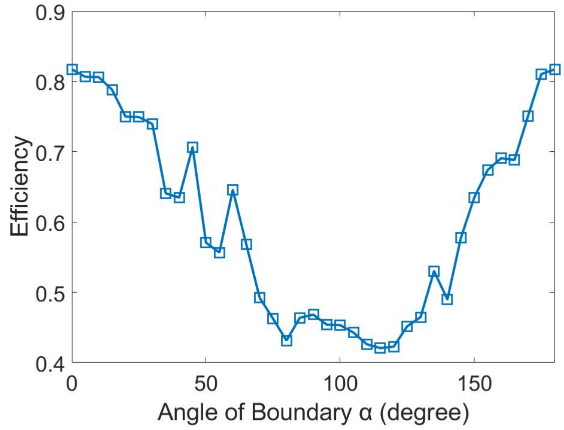

Fig. 10 Efficiency decrease induced by quasi-periodical effect.

V. Conclusion

This paper studied the quasi-periodical effect for the nanorods metasurface. Both theoretical and simulation results show that it may cause phase error in the design if the coupling among elements are simplified with periodic coupling. The inconsistent boundary condition could induce phase error up to 80 degrees for circularly polarized light. Moreover, the efficiency could also be deteriorated up to 40%. We will study the correction method in the future research.

Acknowledgment

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